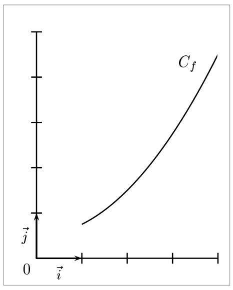
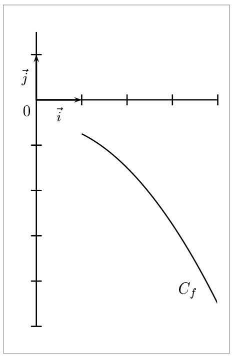
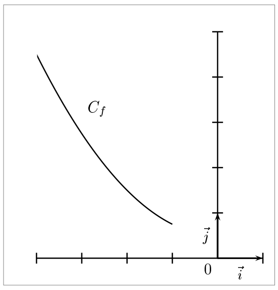


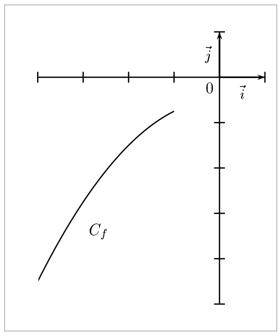
$\lim\limits_{x\to+\infty} f(x) = +\infty$



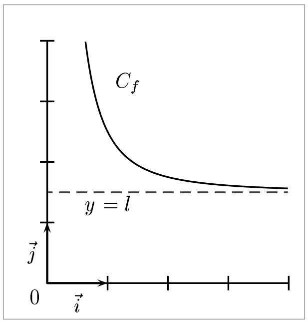
$\lim\limits_{x\to+\infty} f(x) =$



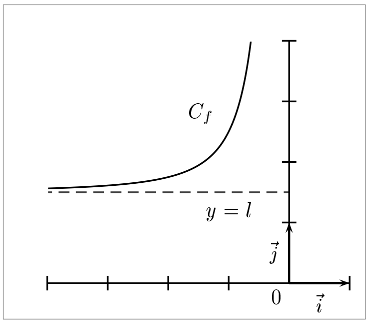
$\lim\limits_{x\to-\infty} f(x) =$



$\lim\limits_{x\to-\infty} f(x) =$

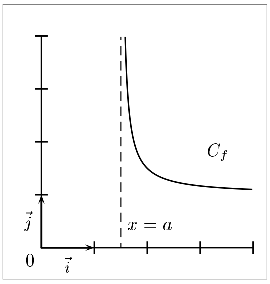


$\lim\limits_{x\to+\infty} f(x) = l$



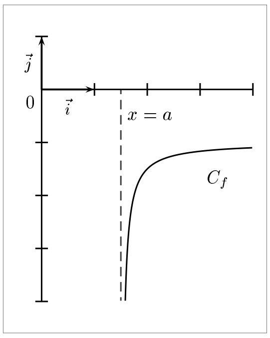
$\lim\limits_{x\to-\infty} f(x) = l$

sur $]a\ ,\ b[$



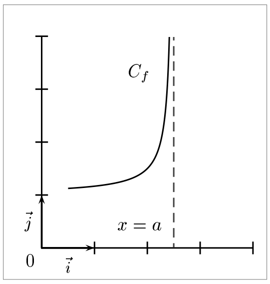
$\lim\limits_{\substack{x\to a \\ x>a}} f(x) = +\infty$

sur $]a\ ,\ b[$



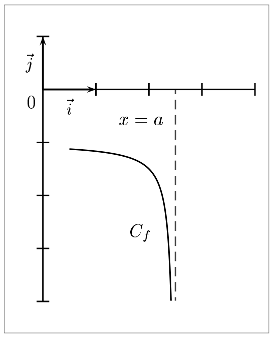
$\lim\limits_{\substack{x\to a \\ x>a}} f(x) =$

sur $]c\ ,\ a[$



$\lim\limits_{\substack{x\to a \\ x<a}} f(x) =$

sur $]c\ ,\ a[$



$\lim\limits_{\substack{x\to a \\ x<a}} f(x) =$

a) Limites des fonctions de références

Propriété

• En $+\infty$:

$\lim\limits_{x\to+\infty} x =$; $\lim\limits_{x\to+\infty} x^2 =$; $\lim\limits_{x\to+\infty} x^3 =$; $\lim\limits_{x\to+\infty} \sqrt{x} =$

$\lim\limits_{x\to+\infty} \dfrac{1}{x} =$; $\lim\limits_{x\to+\infty} \dfrac{1}{x^2} =$; $\lim\limits_{x\to+\infty} \dfrac{1}{x^3} =$; $\lim\limits_{x\to+\infty} \dfrac{1}{\sqrt{x}} =$

Propriété

• En $-\infty$:

$\lim\limits_{x\to-\infty} x =$; $\lim\limits_{x\to-\infty} x^2 =$; $\lim\limits_{x\to-\infty} x^3 =$

$\lim\limits_{x\to-\infty} \dfrac{1}{x} =$; $\lim\limits_{x\to-\infty} \dfrac{1}{x^2} =$; $\lim\limits_{x\to-\infty} \dfrac{1}{x^3} =$

Propriété

• En 0 :

$$\lim_{\substack{x\to 0\\x>0}} \frac{1}{x} = \quad ; \lim_{\substack{x\to 0\\x>0}} \frac{1}{x^2} = \quad ; \lim_{\substack{x\to 0\\x>0}} \frac{1}{x^3} = \quad ; \lim_{\substack{x\to 0\\x>0}} \frac{1}{\sqrt{x}} =$$

$$\lim_{\substack{x\to 0\\x<0}} \frac{1}{x} = \quad ; \lim_{\substack{x\to 0\\x<0}} \frac{1}{x^2} = \quad ; \lim_{\substack{x\to 0\\x<0}} \frac{1}{x^3} =$$

▶ **Remarque :** les fonctions sinus et cosinus n'admettent pas de limite aux infinis.

b) Opérations sur les limites

On note **FI** (pour forme indéterminée) les cas où les théorèmes ne permettent pas de conclure.
l et l' représentent deux nombres finis.

▶ **Limite d'une somme :**

Situation	Exemple
$\underbrace{(\quad)}_{\to l} + \underbrace{(\quad)}_{\to l'} \to$	$\lim\limits_{x\to+\infty} 3 + \left(\dfrac{1}{x}\right) =$
$\underbrace{(\quad)}_{\to l} + \underbrace{(\quad)}_{\to+\infty} \to$	$\lim\limits_{\substack{x\to 0\\x>0}} 4 + \left(\dfrac{1}{x}\right) =$
$\underbrace{(\quad)}_{\to l} + \underbrace{(\quad)}_{\to-\infty} \to$	$\lim\limits_{x\to-\infty} \left(\dfrac{1}{x}\right) + x =$

Situation	Exemple
$\underbrace{(\quad)}_{\to+\infty} + \underbrace{(\quad)}_{\to+\infty} \to$	$\lim\limits_{x\to+\infty} x^2 + \sqrt{x} =$
$\underbrace{(\quad)}_{\to-\infty} + \underbrace{(\quad)}_{\to-\infty} \to$	$\lim\limits_{x\to-\infty} x^3 + x =$
$\underbrace{(\quad)}_{\to+\infty} + \underbrace{(\quad)}_{\to-\infty}$	

▶ **Limite d'une différence :**

• *Exemples :*

$$\lim_{x\to+\infty} \left(\frac{1}{x}\right) - 5 = \qquad \lim_{\substack{x\to 0\\x>0}} \sqrt{x} - \left(\frac{1}{x}\right) =$$

$$\lim_{x\to-\infty} x^2 - x = \qquad \lim_{\substack{x\to 0\\x<0}} \left(\frac{1}{x^2}\right) - \left(\frac{1}{x}\right) =$$

▶ **Limite d'un produit :**

Situation	Exemple
$\underbrace{(\quad)}_{\to l} \times \underbrace{(\quad)}_{\to l'} \to$	$\lim\limits_{x\to+\infty} 4 \times \left[3 + \left(\dfrac{1}{x}\right)\right] =$
$\underbrace{(\quad)}_{\to l>0} \times \underbrace{(\quad)}_{\to+\infty} \to$	$\lim\limits_{\substack{x\to 0\\x>0}} \left(x^2+3\right) \times \left(\dfrac{1}{x}\right) =$
$\underbrace{(\quad)}_{\to l<0} \times \underbrace{(\quad)}_{\to+\infty} \to$	$\lim\limits_{x\to+\infty} \left[\left(\dfrac{1}{x}\right) - 1\right] \times \sqrt{x} =$
$\underbrace{(\quad)}_{\to l>0} \times \underbrace{(\quad)}_{\to-\infty} \to$	$\lim\limits_{\substack{x\to 0\\x<0}} (4+x) \times \left(\dfrac{1}{x}\right) =$

Situation	Exemple
$\underbrace{(\quad)}_{\to l<0} \times \underbrace{(\quad)}_{\to -\infty} \to$	$\lim\limits_{x\to-\infty} \left[\left(\dfrac{1}{x}\right)-5\right] \times x^3 =$
$\underbrace{(\quad)}_{\to +\infty} \times \underbrace{(\quad)}_{\to +\infty} \to$	$\lim\limits_{x\to+\infty} x^2 \times \sqrt{x} =$
$\underbrace{(\quad)}_{\to -\infty} \times \underbrace{(\quad)}_{\to -\infty} \to$	$\lim\limits_{x\to-\infty} x^3 \times \left(7-x^2\right) =$
$\underbrace{(\quad)}_{\to -\infty} \times \underbrace{(\quad)}_{\to +\infty} \to$	$\lim\limits_{x\to+\infty} (8-x) \times \sqrt{x} =$
$\underbrace{(\quad)}_{\to \pm\infty} \times \underbrace{(\quad)}_{\to 0}$	

▶ **Limite de l'inverse :**

Situation	Exemple
$\left(\dfrac{1}{\underbrace{(\quad)}_{\to l\neq 0}}\right) \to$	$\lim\limits_{x\to+\infty} \dfrac{1}{\left(\left(\frac{1}{x}\right)+2\right)} =$
$\left(\dfrac{1}{\underbrace{(\quad)}_{\to \pm\infty}}\right) \to$	$\lim\limits_{x\to-\infty} \dfrac{1}{(x+2)} =$

Situation	Exemple
$\left(\dfrac{1}{\underbrace{(\quad)}_{\to 0^+}}\right) \to$	$\lim\limits_{\substack{x\to 1\\ x>1}} \dfrac{1}{(x-1)} =$
$\left(\dfrac{1}{\underbrace{(\quad)}_{\to 0^-}}\right) \to$	$\lim\limits_{\substack{x\to 1\\ x<1}} \dfrac{1}{(x-1)} =$

▶ **Limite d'un quotient :**

Pour étudier la limite de $\dfrac{f}{g}$, on écrit que $\dfrac{f}{g} = f \times \dfrac{1}{g}$. On détermine alors la limite de f et de $\dfrac{1}{g}$, ce qui permet d'en déduire la limite de $\dfrac{f}{g}$.

• *Exemples :*

$$\lim_{\substack{x\to 2\\ x<2}} \frac{-5}{x-2} = \lim_{\substack{x\to 2\\ x<2}} (-5) \times \left[\frac{1}{(x-2)}\right] =$$

$$\lim_{\substack{x\to 0\\ x>0}} \frac{x+1}{\sqrt{x}} = \lim_{\substack{x\to 0\\ x>0}} (x+1) \times \left[\frac{1}{(\sqrt{x})}\right] =$$

$$\lim_{\substack{x\to -1\\ x>-1}} \frac{x^2+3}{x+1} = \lim_{\substack{x\to -1\\ x>-1}} (x^2+3) \times \left[\frac{1}{(x+1)}\right] =$$

$$\lim_{x\to-\infty} \frac{2x^3}{1+\frac{1}{x}} = \lim_{x\to-\infty} (2x^3) \times \left[\frac{1}{\left(1+\left(\frac{1}{x}\right)\right)}\right] =$$

$$\lim_{x\to+\infty} \frac{3+\frac{1}{x}}{\sqrt{x}} =$$